Origins of pressure differences around a sail

Alan Smith, BSc.

Sydney, Australia

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smithalan@bigpond.com

Abstract

Sailboats sail upwind. They use a force, the aerodynamic force, which beneficially exploits the relative motion of the sail through air in a manner that is variously described as mysterious, little understood or so complex that the explanation appears to require a deep understanding of fluid dynamics.

It is well understood that the aerodynamic force derives from pressure differences on each side of the sail, whether it be the low pressure pull to leeward or the high pressure push from windward. However, a review of the literature reveals that existing theories about the source of those pressure differences are flawed or inadequate.

This paper asserts that the pressure differences are caused only by the relative movement of the sail exerting pressure on air in front of it, and leaving a region of low pressure behind it.

The paper provides experimental evidence that the pressure differences are sufficient to completely account for the aerodynamic force. Analysis of independent experimental work confirms that the pressure differences account for the forces measured.

In addition, it asserts the novel position that both of these pressure differences can be explained without recourse to an increase or decrease in flow speed.

Principles from thermodynamics are used to demonstrate the characteristics of the pressure differences.

Some numerical analysis of the thermodynamic solution provides encouraging support for the novel explanation of the origin of the pressure differences.

Introduction

Sailboats come in many forms: from 3 metre dinghies, 10 metre club keelboats, 30 metre super-maxies to 100 metre mega sailing yachts. They come with one, two, or many sails made of thin, strong, flexible fabric, supported by one, two, three or more masts.

They all come with a particular facility that is exhibited by even the simplest, single-sailed craft such as a Finn Dinghy or a Laser: they can sail upwind.

They cannot sail directly upwind. No sailboat can sail directly into the wind, but, given two points, A and B in open water, where B is upwind from A, then, without the assistance of tide or current, all sailboats can, by following a zig-zag path, sail from point A to point B.

The challenge for a racing sailor on a course that inevitably contains such upwind legs, is to get from A to B quicker than the competitors.

As an amateur racing sailor the author has learnt many principles of sail and rig trim that are effective in racing. Poorly trimmed sails or poorly tuned rigs make the boat go slower. Better trimmed sails make the boat go faster.

Our instruments can provide feedback by providing Log speed, Apparent Wind Angle (AWA), True Wind Angle (TWA), Velocity Made Good (VMG) etc., but their reliability is dependent on being correctly calibrated and our ability to interpret the numbers.

We make various adjustments and the boat either goes faster, or slower, but we never really know why the adjustment yielded the results.

More often than not the answer will be couched in terms like laminar flow, boundary layers, laminar separation, viscosity, vortices, aerofoil shape etc., all of which come from fluid dynamics, which are totally obscure to the layman sailor and which don't really address the question of when and by how much to apply an adjustment.

The literature of sailing abounds with the classical diagram of the forces on a sailing boat and Fig.1 has been taken as the primary source from Fossati [5] .

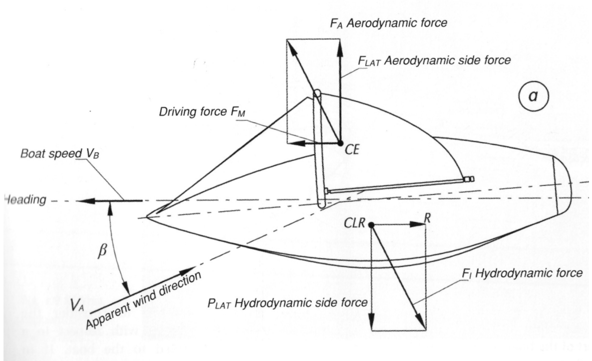


Fig.1 forces on a sailing boat

The arrows in the diagram indicate the size and direction of the forces acting on the boat.

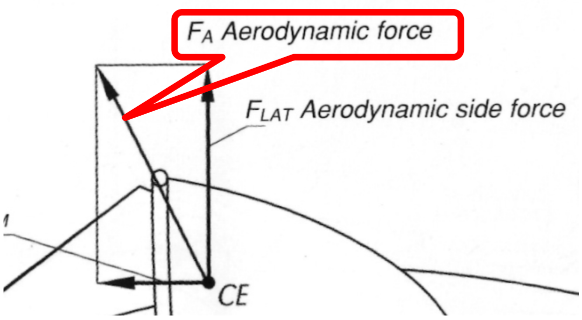


Fig.2 Aerodynamic Force

Most of the forces in the diagram were well explained in Fossati's book but the explanation for "F_A Aerodynamic Force" was elusive. From basic physics it is well known that the only ways in which fluid can exert a force on a solid are by applying a pressure over an area, and through friction between the fluid and the solid. Since air is a gas, and a gas is a fluid, and the sail is a solid (if flexible) object, the aerodynamic force has to come from pressure and/or shear stress.

It is also well known that, like friction, shear stress is a dissipative force that converts mechanical energy into thermal energy due to the generation of heat by the interaction between the fluid and the solid surface, so it is highly unlikely that any shear stress arising from the flow of air over the sail would make a positive contribution to the aerodynamic force.

So a search was undertaken to identify the nature and source of the pressure differences around a sail that are responsible for the creation of the aerodynamic force.

Literature review

While sailing books may not always fall into the category of scholarly works, many of them approach the subject in a scholarly manner. Sailing is a complex and technical subject that requires a deep understanding of various topics, including navigation, meteorology, sail theory, and boat design. As a result, many sailing books are written by experts in these fields who have conducted extensive research and have a wealth of practical experience.

In the search for an explanation of the source of the pressure difference, recommendations were sought from various sources including sailing acquaintances, on-line forums and references. Given the number of titles, the list is by no means comprehensive but there was no deliberate bias in the selection.

What follows is a brief review of the explanation of the source of the pressure variations presented in a sample of the sailing literature about the source of pressure differences around a sail.

Sailing Literature

Fossati [5]

Fossati's book, "*Aero-hydrodynamics and the performance of sailing yachts"*, epitomises many of the approaches taken to explain the source of the aerodynamic force.

He is cited in a number of research papers on the aerodynamics of sails by Prof. Richard Flay of Auckland University.

A professor of applied mechanics, Fabio Fossati was selected as a primary source of technical sailing literature as he taught fluid mechanics, naval architecture and mechanics of the sailing yacht on the Master's course in Yacht Design at Milan Polytechnic. He was Research Associate of the International Technical Committee of the Offshore Racing Congress and also a keen sailor.

In chapter 5 "Sailing Boat Aerodynamics Para 5.1 "The aerodynamics of the sail", p. 92, Fossati asserts that::

" …when a sail deflects and affects airflow, because of the link between flow speed and pressure (see Appendix 1) an area of weak high pressure is created to windward and a suction area is created to leeward of the sail."

In Appendix 1, "Elements of Fluid Mechanics", the existence of these high and low pressure regions is clearly demonstrated experimentally in fluids, both liquid and gas, flowing through tubes and around foil sections.

The existence of these pressure variations is not a revelation.The source of these pressure variations is attributed as follows:

"These pressure variations are attributable to a variation in dynamic pressure induced by a flow speed variation. It is in fact evident that where the section where the flow passes narrows, because of the conservation of fluid flow, there must be an acceleration of the flow which must correspond with a reduction in pressure."

It is the circular nature of this argument that is at the heart of the mystery: on p. 92, the deflection of the airflow by the sail influencing the flow speed is the source of the pressure variations, and now in Appendix 1, we find that the pressure variations must be due to an acceleration of the flow.

In the first, the pressure variations due to flow speed and in the second, the variation in flow speed is due to pressure variations.

Bernoulli's Law is invoked to explain the link between the accelerating flow and the pressure variations and from there, the fluid dynamics arguments are based on the existence of an accelerated airflow.

Fossati presents no evidence to support the existence of an accelerated airflow over an aerofoil.

Whidden and Levitt [9]

Tom Whidden is president and CEO of North Technology Group, which includes North Sails. Whidden is an America's Cup sailor who has won the Cup three times: 1980,1986-87, and 1988, primarily as tactician for Dennis Conner. Whidden is in the America's Cup Hall of Fame. He is the author of three books on sails and sailing including "*The Art and Science of Sails Revised Edition"* which is referenced below.

p. 57:

'*A sail is like an airplane wing.' If you missed that lesson early in your sailing career, you likely heard its antecedent in sixth-grade science: An airplane flies because air passing over the curved upper surface of its wings has to travel a longer distance than the air passing under the flat lower surface. And since it has to go farther, it has to go faster to reach the trailing edge at the same time as its brother particle. This difference in distance causes a difference in speed that causes a difference in pressure-low pressure on the upper side of the wing (lee side of the sail) and high pressure on the lower side of a wing (wind-ward side of a sail)"*.

This is an example of the "Equal Transit Time " explanation, a common misconception which is often used to explain the increase of the speed of the air speed flowing over a wing or a sail as required by Bernoulli's Principle to generate a low pressure.

Jobson [6]

Gary Jobson has won ten national one-design sailing titles, the America's Cup, and innumerable ocean races. In 1999 he received the U.S. Sailing Association's most prestigious award, the Nathanael G. Herreshoff Trophy. In 2003 Gary was inducted into the America's Cup Hall of Fame. He is an editor at large for Cruising World and Sailing World and has been ESPN's sailing commentator since 1985.His book "*Championship Sailing* " is referenced b below:

P. 166 :
"*No matter what sail you may be flying - mainsail, genoa, spinnaker, or staysail - they all work on the same principle: wind bends around the sail, causing a vacuum on the forward leeward side. The boat moves forward to fill this vacuum."*

This is an interesting view but, unfortunately he takes this no further to explain the nature of this phenomenon or to refer to the existence or significance of the high pressure to windward.

Dedekam [3]

Ivar Dedekam has no great CV but has written a small but highly regarded handbook, "*Illustrated Sail & Rig Tuning"*.

p. 3:

"*The same happens when air flows along a sail (or an airplane wing). The shape of the sail forces the airflow on the leeward side to take a longer path than on the windward side. Therefore the air has to increase its velocity on the leeward side of the sail resulting in a lower pressure than on the windward side. (Bernoulli's principle states that an increase of velocity in a fluid flow gives a pressure decrease.)*"

Once again the disproved "equal transit time" theory is used to explain the pressure variation.

Melges[7]

Harry C. "Buddy" Melges Jr. is a competitive sailor. He has earned national and international championships in several classes in conventional sailing and ice-boating. His book "*Sailing Smart"* is referenced below:

P. 73:

"*Every sail gets its driving force from its shape, and this power is controlled by the amount and location of a sail's draft, or camber. Never allow the air flow to stall or separate from the sail's surface (unless you become overpowered) because that reduces the driving force of the sail. Always keep in mind that it is the amount and the velocity of the air that flows across the underside of your sails that determine how much power you are going to get from those sails. Maintaining this smooth flow applies to all points of sail: on the wind as well as running free.*"

Simple and direct: it's the shape of the sail that provides the driving force. However, there is no attempt to explain how this occurs.

Marchaj [8]

C.A. Marchaj was a visiting research fellow at the Department of Aeronautics and Astronautics, Southampton, England, where he held a Master's Degree in the Faculty of Engineering and Applied Science. A former Polish champion of the Finn class, he was a chartered engineer, a glider pilot, and a member of the Royal Institution of Naval Architects. His book, "*Aero-Hydrodynamics of Sailing"* is widely referenced as an authority on the subject.

P. 180:

"*A little reflection however shows that Bernoulli's theorem locates the region of higher pressure in places where the free motion of fluid is retarded. Since pressure may be regarded as a form of energy and Bernoulli's equation indicates that a balance is maintained between energy arising from the motion and that from the pressure in all parts of the stream; it becomes rather obvious and inevitable law that what has been lost in one form of energy must be recovered in another form (Ref 2.5)*."

Marchaj ascribes the pressure difference to a velocity difference but fails to explain the source of the velocity difference.

Aerodynamics literature

Since these sailing books failed to deliver a satisfactory answer and many of them invoked aerodynamics or description of a sail being like a wing, a search of the aerodynamics literature was undertaken.

However, after an extensive search of the literature, none could be found that discussed sails or provided any information about the source of the pressure differences around a sail.

Summary

Whilst this is by no means an exhaustive review of the available literature, and indeed is a small subset of the books, articles and websites that have been reviewed, it is a representative cross-section of the way that the source of the pressure variation is addressed.

The sailing literature leaves a gap in the explanation of the source of the pressure difference which is not filled from the aerodynamics literature.

It is apparent that there is no consensus on the subject and that there is no satisfactory documented explanation for the source of the pressure difference.

Hypothesis

The hypothesis is that the pressure variations are caused only by the movement of the sail through the air, compressing the air to windward and leaving a relative void to leeward.

This hypothesis asserts that pressure differences, both increased and decreased, can be generated in air without reference to any associated increase or decrease in flow speed.

The hypothesis is tested using:

- an alternative frame of reference with the sail moving in still air rather than the wind blowing over the sail
- a thought experiment using a foam of idealised soap bubbles to represent the volume of air through which the sail moves.

Frame of Reference

It is well established that a sailing yacht is able to sail with the wind blowing from a direction ahead, so the explanation commences with Fig. 3, a diagram of a yacht sailing upwind. The apparent wind angle is the angle between the sail chord and the apparent wind direction. (Fossati [5]).

To develop the theory, the image has been flipped and rotated in Fig. 4 to bring the apparent wind direction to the horizontal.

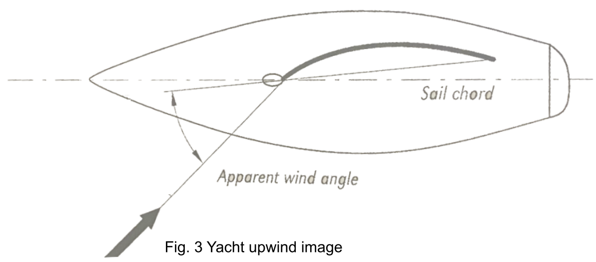


Fig. 3 Yacht upwind image

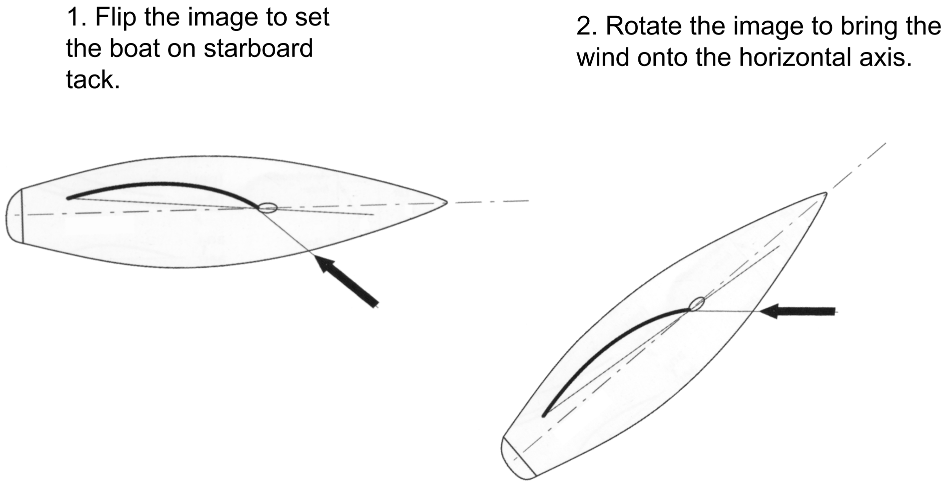


Fig 4. Transposed image

The sail is now considered in the frame of reference of the apparent wind, with the sail moving through the wind instead of the wind moving over the sail.

In either frame of reference, the laws of physics apply. Strictly speaking, as long as the frames of reference are not accelerating, any one frame of reference is as good as another.

In this new frame of reference, the sail is moving in relation to the stationary wind instead of the wind moving in relation to the stationary sail. Fig. 5

And since, in this frame of reference, the wind is stationary, we will simply refer to it as air.

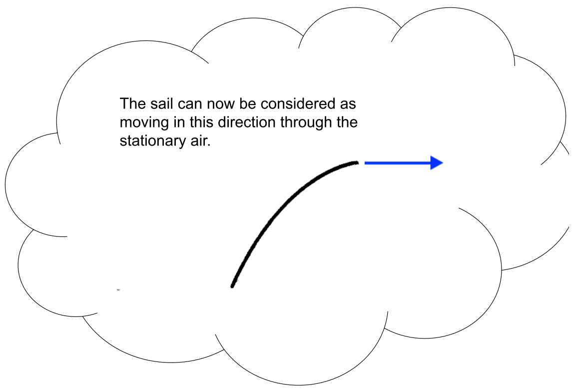


Fig. 5 Sail moving relative to still air

Let's now consider what happens as the sail moves through the air from position A to position B. Fig 6.

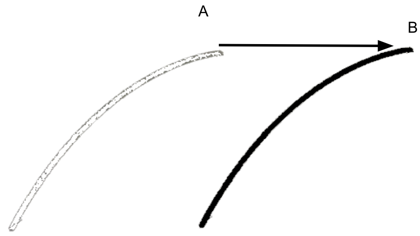


Fig. 6 Sail moving A to B

Let's call the area between the sail at the two positions the "swept area", as like a broom, the sail sweeps over the area. Fig. 7

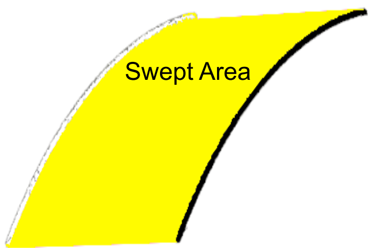


Fig. 7 Swept Area

And let's not forget that we have been looking at a 2-dimensional representation, i.e. a cross section of the sail Fig 8. (Fossati [6]).

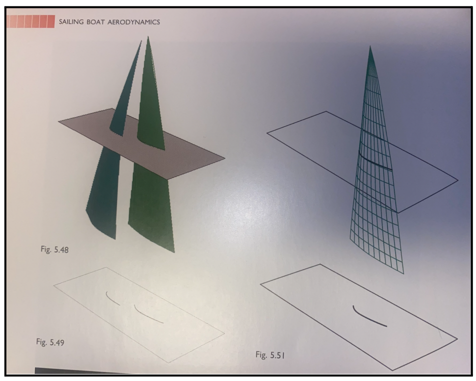


Fig. 8. Cross section through the sail

The swept area represents the cross section through a **volume** of air, and so we will refer to it as the "swept volume". Fig. 9

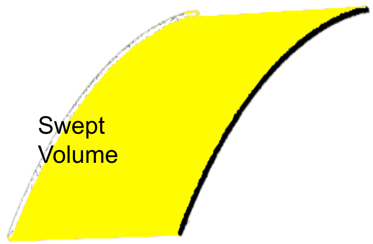


Fig. 9. Swept Volume

Thought Experiment

You are now invited to participate in a "thought experiment" in which the air through which the yacht is sailing is filled with small soap bubbles. This is not an ordinary foam of soap bubbles of different sizes, whose film is made up of soapy water which is fragile and wet, but one that consists of equal sized idealised bubbles whose film is composed of a material that is thin, strong, light and infinitely flexible. This film also exhibits the same characteristic that is shared by all liquids including the soapy water that creates bubbles, namely surface tension that binds the film and provides the pressure that causes free-floating bubbles to take on the familiar spherical shape that encloses a given volume with the smallest surface area.

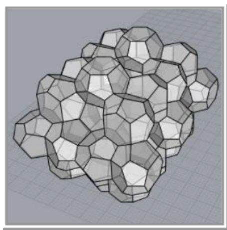


Fig 10 Tessellated polyhedra

When bubbles pack together, like in foam, the space will be filled with bubbles and surprisingly a lot of research has been done to investigate the shape these bubbles will form. Searching "Weaire Phelan structure" will take you down a rabbit hole of space-filling convex polyhedra and Kelvin structures, like in Fig 10. but suffice it to say that the bubbles so formed will not be the familiar spheres of free-floating bubbles. If the bubbles are of equal size, they will form a mesh of polyhedrons with flat faces, and straight sides.

In the following figures, the bubbles are represented by circles and represent a cross-section through a 3-dimensional region.

In Fig 11, the bubbles in the swept volume have been coloured red. These are the bubbles that occupy the region swept by the sail as it moves from position A to position B

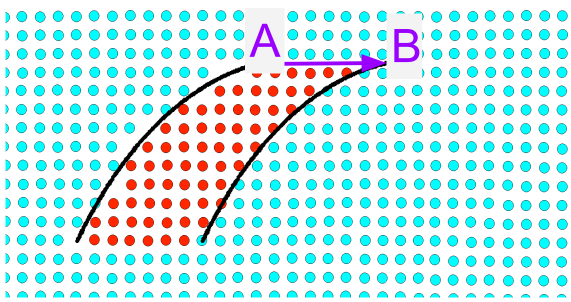


Fig. 11 Red bubbles in swept volume

All the red bubbles are swept by the sail into the space that was previously occupied by cyan bubbles. We know that two bubbles are unable to occupy the same space and time, so they must become compressed and flow out into the region ahead, forcing most of the bubbles in the region ahead to also become compressed.The bubbles in the swept volume become compressed along with the bubbles that previously occupied the space as well as some of the bubbles further upwind.

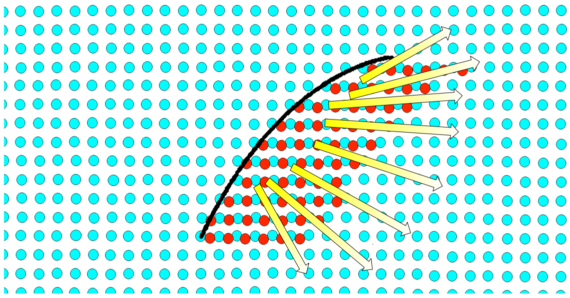


Fig.12 Compressed red bubbles

Eventually, as the bubbles are pushed further outwards they become decreasingly compressed until at some distance to windward, their pressure difference from the original bubbles becomes insignificant.

Attention is now turned to the swept volume to leeward or behind the sail in Fig. 13. It is apparent that bubbles in the region behind the sail will now have to move in to fill the void that is left as the sail moves forward. Since bubbles can't be created to fill this void, they are drawn in from the surrounding region, so the same number of bubbles will now have to occupy a larger volume. With the same amount of air filling a larger bubble, these bubbles will now experience air pressure lower than the surrounding region.

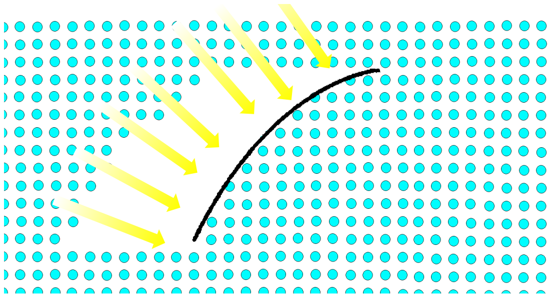


Fig. 13 Swept volume to leeward.

So now this region of high pressure can be seen in front of the sail, with this region of low pressure behind the sail. Fig. 14

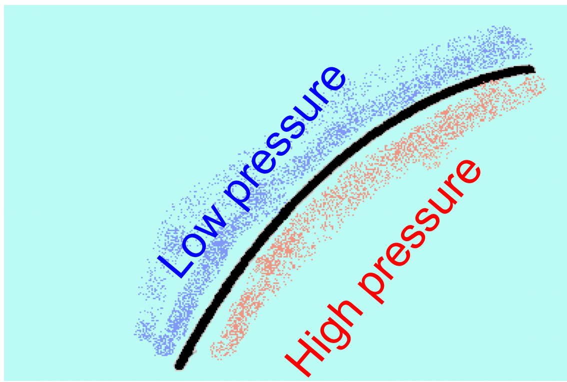


Fig. 14 pressure distribution

The pressure regions

The question now arises as to the characteristics of these high and low pressure regions: their size, shape and intensity and leads the discussion into the consideration of thermodynamics. Whilst thermodynamics is a general science that deals with the relationships between heat, work, temperature, and energy, its application to real-world problems is primarily through the study of the nature of gases, such as the air around the sail in which the pressure regions occur.

The behaviour of gases is governed by the ideal gas law, which states that the pressure of a gas is proportional to its temperature and inversely proportional to its volume. This law can be used to predict the behaviour of gases in a variety of situations, such as in engines, refrigerators, and air conditioners.

In addition to the ideal gas law, thermodynamics also provides insights into the behaviour of other types of systems, such as liquids and solids. However, the study of gases is often the most straightforward way to apply thermodynamics to real-world problems.

The ideal gas law can be used to calculate the pressure, temperature, and volume of a gas.

The kinetic theory of gases can be used to explain the behaviour of gases at the molecular level.

The ideal gas law relates the pressure, temperature and volume of a quantity of gas through the equation PV=KT, where

- P is the pressure of the gas
- V is its volume,
- T is it temperature
- and K relates to the amount of the gas under consideration.[1]

The ideal gas law is a simplified model of the behaviour of gases. It assumes that gas molecules have no volume and do not interact with each other. However, air molecules do have a small volume and do interact with each other.

This deviation from the ideal gas law is negligible at the temperatures and pressures experienced around a yacht's sail[2].

The Volume challenge

The challenge in applying thermodynamics to the problem of the sail is that whilst the pressure (P) and temperature (T) of the air around the sail is quite easily identified, the question of volume (V) presents a challenge to the use of thermodynamics in this situation.

However, consideration of the speed with which a disturbance in the pressure, temperature or density propagates through the air makes the problem less intractable. That speed is, of course, the speed of sound in the air.

So the passage of the sail through the air is influenced by the pressure in the air **that has been disturbed** by its passage.

Consider the following Fig 15.

[1] PV = nRT is the usual formulation, but since R is the Gas Constant and "n" is the number of moles of gas under consideration, their combination into a single variable K related to the amount of gas is accepted practice.

[2] Cengel [2] p. 726: "The temperature of air in air-conditioning applications ranges from about -10 to about 50°C. In this range, dry air can be treated as an ideal gas with a constant cp value of 1.005 kJ/kg·K [0.240 Btu/lbm·R] with negligible error (under 0.2 percent). … The atmospheric air can be treated as an ideal-gas mixture whose pressure is the sum of the partial pressure of dry air P_a and that of water vapour P_v"

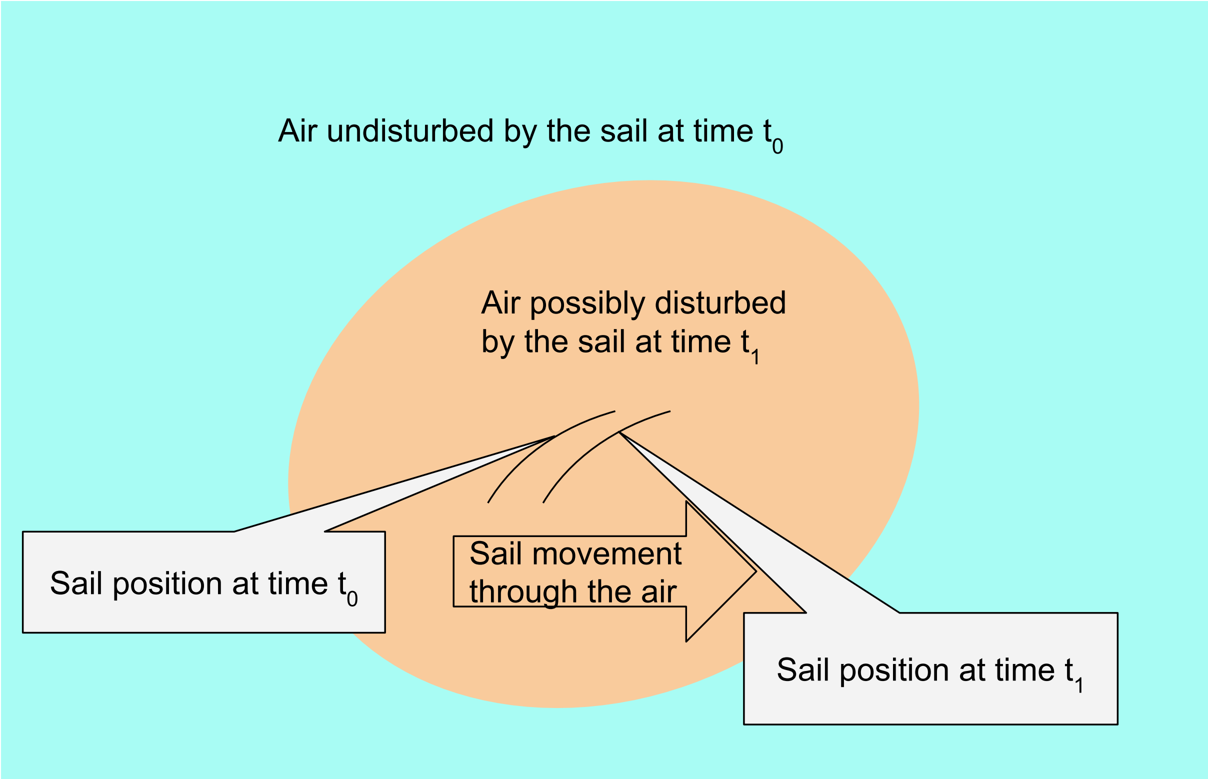


Figure 15. Region of air disturbed by the sail

At any particular instant in time, t_1, there is a region beyond which the speed of sound prevents any pressure, density or temperature disturbance from the passage of the sail through the air at time t_0. The boundary of the possibly disturbed air is determined by the distance that the disturbance can travel from the sail in time t_1-t_0 which we shall call "delta t" or Δt.

The distance is shown as the equal-length dotted arrows in Fig 16 which shows the boundary around the region.

The length of the arrows is c_s x Δt, where c_s is the speed of sound in the undisturbed air.

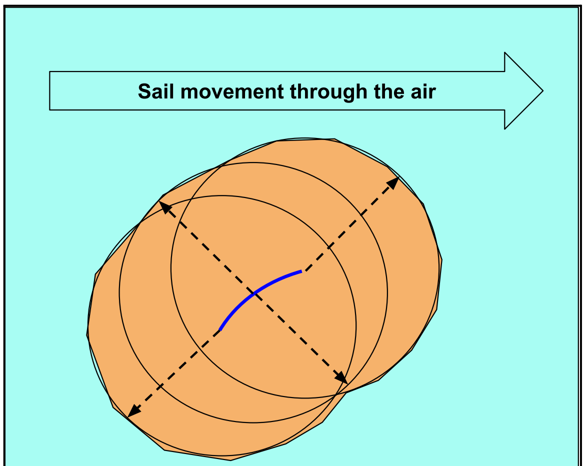


Figure 16. Boundary of disturbed region.

It should be noted that the above figures are not intended to indicate any scale, since the speed of the sail through the air is much lower than the speed of sound in air. A 20 kt apparent wind speed (typical of a yacht doing about 7 kts upwind in a 15 kt wind) relates to about Mach 0.03, or .03 times the speed of sound.

Nor does the colouring of the region indicate anything about the distribution of pressure, temperature or density within the region. These features are discussed in the following section.

It should also be noted that this is a 2-D representation - effectively a horizontal cross-section through the sail.

The piston analogy

To explore the way that the sail generates the pressure differences, we return to Fig 14 and consider the long narrow cylindrical region that passes through the sail as visualised in Fig 17.

The ends of the tube are open to the region of air that is un disturbed around the sail as discussed in the previous section and the section of sail can be considered as a piston moving through this cylinder.

Before addressing the obvious issues of the angle of the cylinder to the sail and the direction of the sail through the air, and its relationship with the disturbed air adjacent to the sides of the cylinder, its consideration from a thermodynamics perspective will yield an valuable and unexpected feature which is explored below.

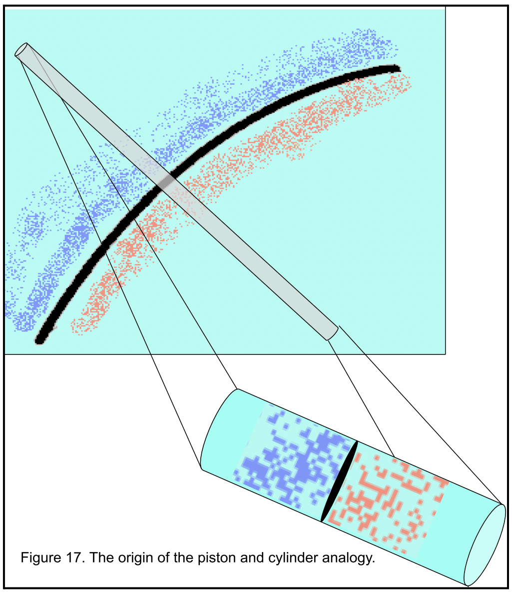


Figure 17. The origin of the piston and cylinder analogy.

A piston pushing a fluid through a cylinder is a construction that is widely studied in the literature of thermodynamics, particularly when discussing the speed of sound and Mach number.

As the piston progresses through the fluid, a pressure wave is generated that has a well defined wave front (Fig 18.) where there is an abrupt change in the pressure and density of the fluid, and this wave front propagates at the speed of sound in the stationary fluid. Cengel [2] p. 843

It is important to recognise that this wave front is not a "shock wave" generated by a supersonic aircraft or speeding bullet, because this pressure wave will be generated by a piston, or the sail, that is moving through the air at well below the speed of sound. The 20 kt wind speed used in Eiffel's experiments and experienced while sailing equates to about 3% of the speed of sound, or Mach 0.03.

Consider the following example of a piston moving in a long tube open at each end to the undisturbed air described above. The piston is shown in Fig 19 below at time t_0 and at a short time later, at time t_1.

The air ahead of the piston is being compressed and the air behind the piston is being expanded by the movement of the piston.

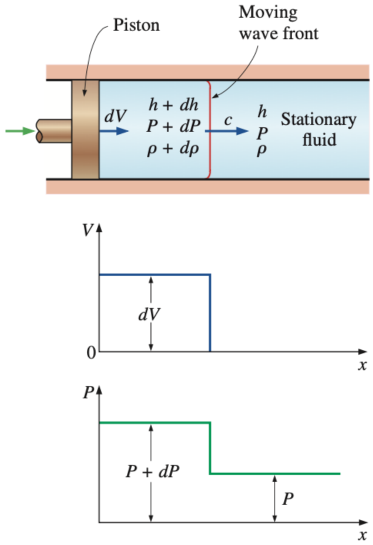


Figure 18. Example of a cylinder and piston used in thermodynamics. Cengel[2] p 843

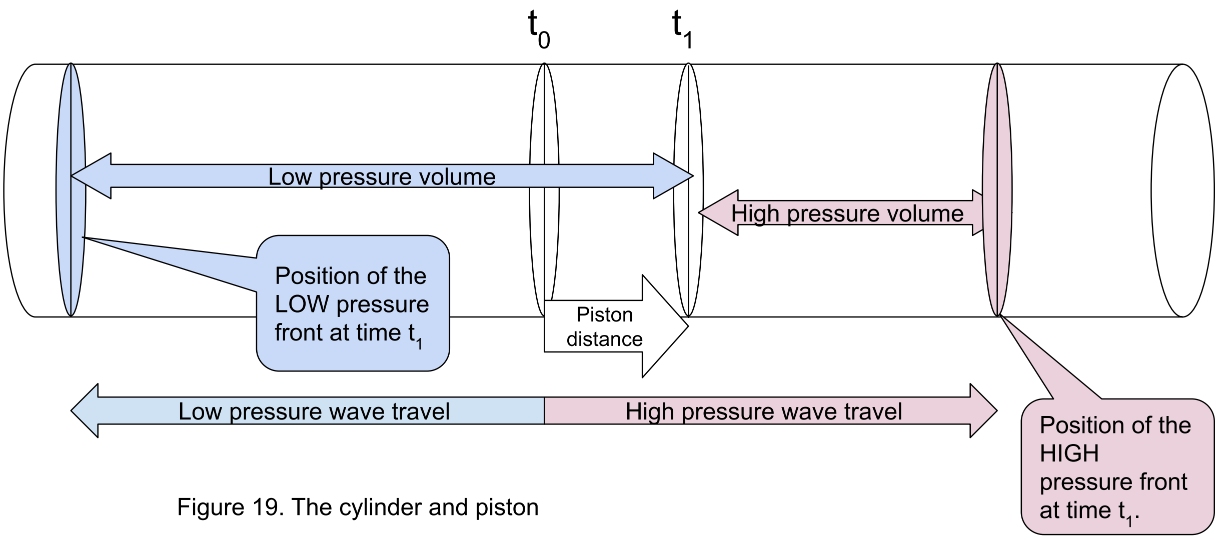


Figure 19. The cylinder and piston

We can calculate the pressure ahead of the piston moving at speed c_P, after time Δt using the adiabatic process as follows:

In an adiabatic expansion $pV^{\gamma} = \ constant$ Blundel[1]l p. 117 (1)

Where

p is the pressure of the gas

V is the volume of the gas

γ (gamma) is the adiabatic constant $(\frac{C_p}{C_V})$which for air is 1.4

So when a quantity of gas is subjected to an adiabatic expansion, from pressure p_0 and volume V_0 to pressure p_1 and volume V_1, we have

$$p_1 V_1^{\gamma} = p_0 V_0^{\gamma} \qquad (2)$$

p_0 is the known undisturbed air pressure

V_0 is the the cross-section area of the piston (A) x the distance the pressure front travels in time t, so

$$V_0 = A\, c_s \Delta t \qquad (3)$$

Where

A is the cross-section area of the piston

c_S is the speed of sound in the undisturbed air

Δt is the time interval

Substituting (3) into (1)

$$p_0 V_0^{\gamma} = \text{constant} = p_0 (A\, c_s \Delta t)^{\gamma} \qquad (4)$$

V_1 is V_0 minus the volume the piston sweeps through in time Δt,

so $V_1 = V_0 - A\, c_P\, \Delta t$ (5)

Substituting V_0 from (3) into (5)

$$V_1 = A\, \Delta t\, (c_s - c_P) \qquad (6)$$

Substituting V_0 from (3) and V_1 from (6) into (2)

$$p_1 = \frac{p_0(A\, c_s \Delta t)^{\gamma}}{(A\, \Delta t\, (c_s - c_p)^{\gamma}} \qquad (7)$$

The $(A\, \Delta t)^{\gamma}$ term cancels out top and bottom yielding:

$$p_1 = p_0\left(\frac{c_S}{(c_s - c_p)}\right)^{\gamma} \qquad (8)$$

A simple examination of the (c_s - c_p) term relating to the volume ahead of the piston for will demonstrate that the pressure **behind** the piston is given by:

$$p_1 = p_0\left(\frac{c_S}{(c_s + c_p)}\right)^{\gamma} \qquad (9)$$

This means that the pressure in front and behind the pistons is **independent of the time interval,** Δt, over which the pressure is measured.

This is a significant conclusion because it demonstrates that the pressure generated by the movement of the piston can be derived directly from the speed of the piston relative to the speed of sound in the undisturbed air whose pressure is known.

The "volume challenge" problem has been resolved.

Temperature

Since thermodynamics has been invoked to develop the argument, it is appropriate to investigate the temperature implications in this adiabatic process, in which

$p^{\gamma-1}T^{\gamma} = constant$[3]

so $p_1^{\gamma-1}T_1^{\gamma} = p_0^{\gamma-1}T_0^{\gamma}$

thus $T_1^{\gamma} = \frac{p_0^{\gamma-1}T_0^{\gamma}}{p_1^{\gamma-1}}$ and so $T_1 = \frac{(p_0^{\gamma-1}T_0^{\gamma})}{p_1^{\gamma-1}}^{\frac{1}{\gamma}}$

Which simplifies[4] to $T_1 = T_0\left(\frac{p_1}{p_0}\right)^{(1-\frac{1}{\gamma})}$ (10)

Since we can set p_0 to the normal atmospheric pressure, (9) can be used to determine p_1, we can set T_0 to a normal atmospheric temperature, and use (10) to determine the temperature T_1

These equations are used in the section "Do the numbers add up" on p.18, to provide real-world results.

[3] Blundell [1] p. 117

[4] These equations may appear mathematically daunting, but spreadsheets such as Excel and Google Sheets provide all the mathematical operators required. The thermodynamicists of the early 20th century would have had to solve them with log tables.

Is the pressure differential sufficient to account for the aerodynamic force?

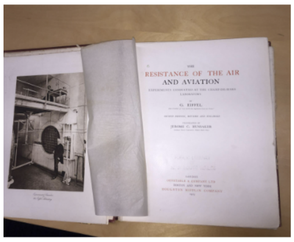


Figure 20 - Eiffel's book

In the search of available literature for experimental data relating to the pressures and forces generated by air over a sail, the work of Gustave Eiffel provided a surprisingly relevant source.

Gustave Eiffel was a pioneer in the field of aerodynamics and his wind-tunnel testing helped to advance the understanding of how air flows around objects. In 1913 Eiffel published his methods and results in a book called "Resistance of the Air and Aviation"[4] It includes detailed descriptions of his wind tunnel, his methods of testing and his results.

The results of his experiments on one particular test subject, the 1/13.5 cambered plate are reported in great detail.

He undertook two sets of measurements on the same plate at the same angle of incidence and speed ranges, the results of which were summarised graphically and are reproduced in Fig. 21 for interest.

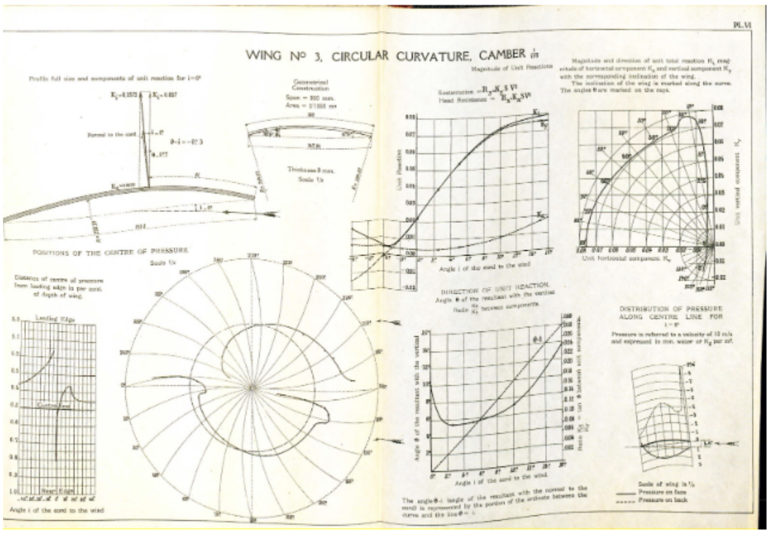


Figure 21 Results of test on 1/13.5 cambered plate

The first experiments used his ingeniously designed apparatus to measure the forces generated by the wind over the plate.

These are the results of the force measurements on the 1:13.5 cambered plate reported on pp. 24,25 reproduced in Fig 22.

The results of the 20° inclination test are highlighted and significance of the values of 0.330 and 0.910 kg weight becomes apparent below.

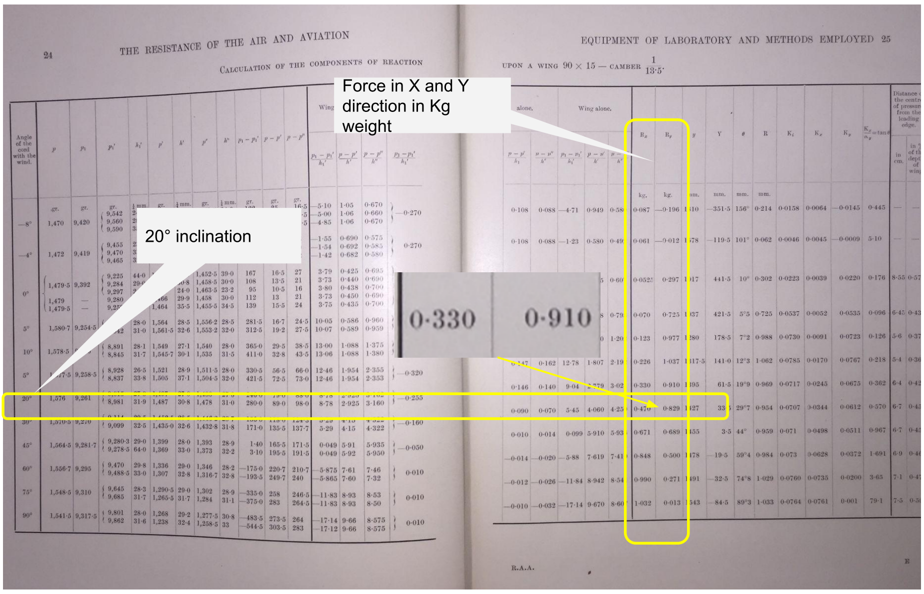


Figure 22. Test results with the 20° inclination vertical (X) and horizontal (Y) force values in Kg weight highlighted.

Quite separately, the **pressure distribution** over the upper and lower surfaces of the plate were measured and reported on "Plate XXV" as isobars calibrated in mmHg above or below atmospheric pressure, reproduced in Fig 23.

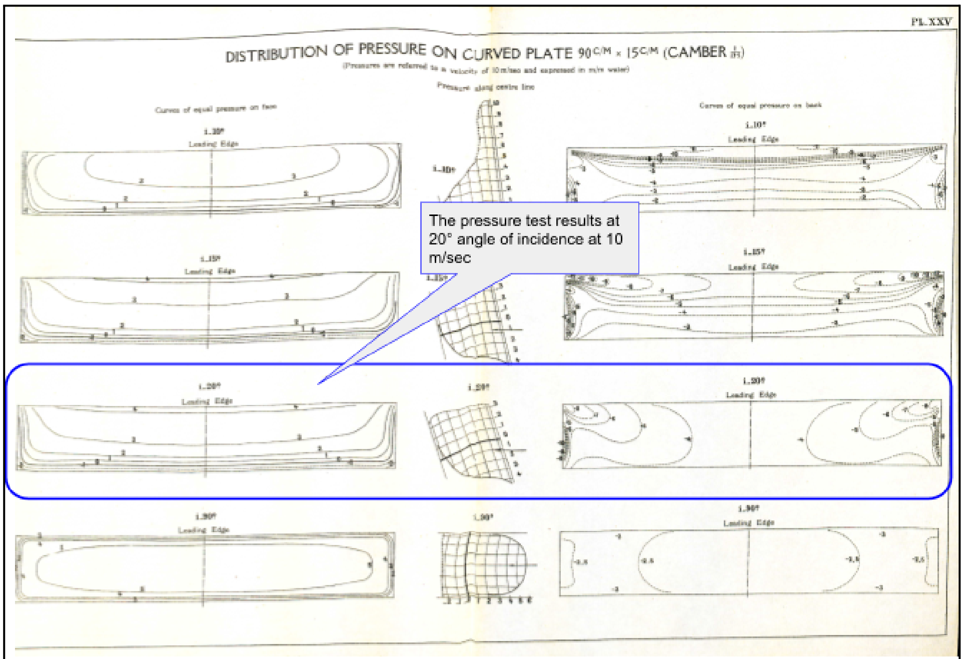


Figure 23. Reproduction of the pressure test results of the 1/13.5 cambered plate

These data are of particular interest for the following reasons:

1. The test subject was a thin plate, 90 x 15 cm, curved into a circular arc which bears a striking similarity to the profile of the sail in Fig. 3.
2. The 10 metres per second air speed used in the experiments corresponds closely to the apparent wind speed encountered while sailing, since 10 metres per second equates to 19.4 knots which would be experienced on a yacht beating upwind at around 7 knots into a true wind of around 15 knots.
3. The angle of incidence of 20° was in the range of apparent wind angle experienced in sailing, in the 20 - 30° range

Analysis

When the pressure data from the 20° incidence test of Fig. 18 above were digitised and analysed it was demonstrated that the forces **calculated** from these pressures mathematically correspond to the forces **measured**.

Since the diagrams are symmetrical around the vertical centre axis, only one side was required to be digitised as illustrated in Fig 24 below:

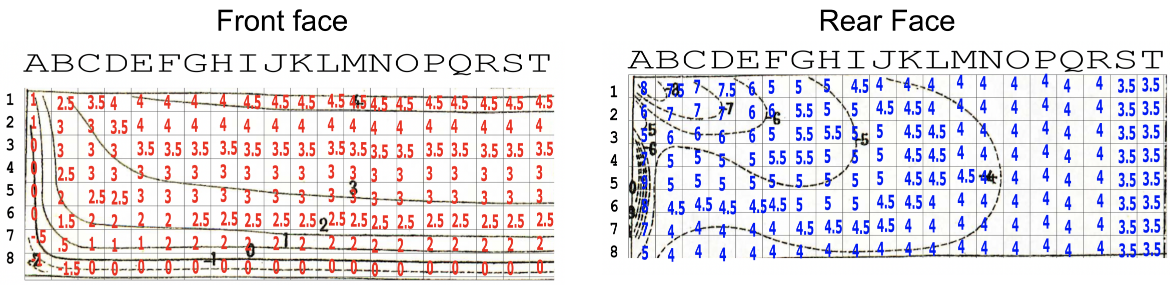
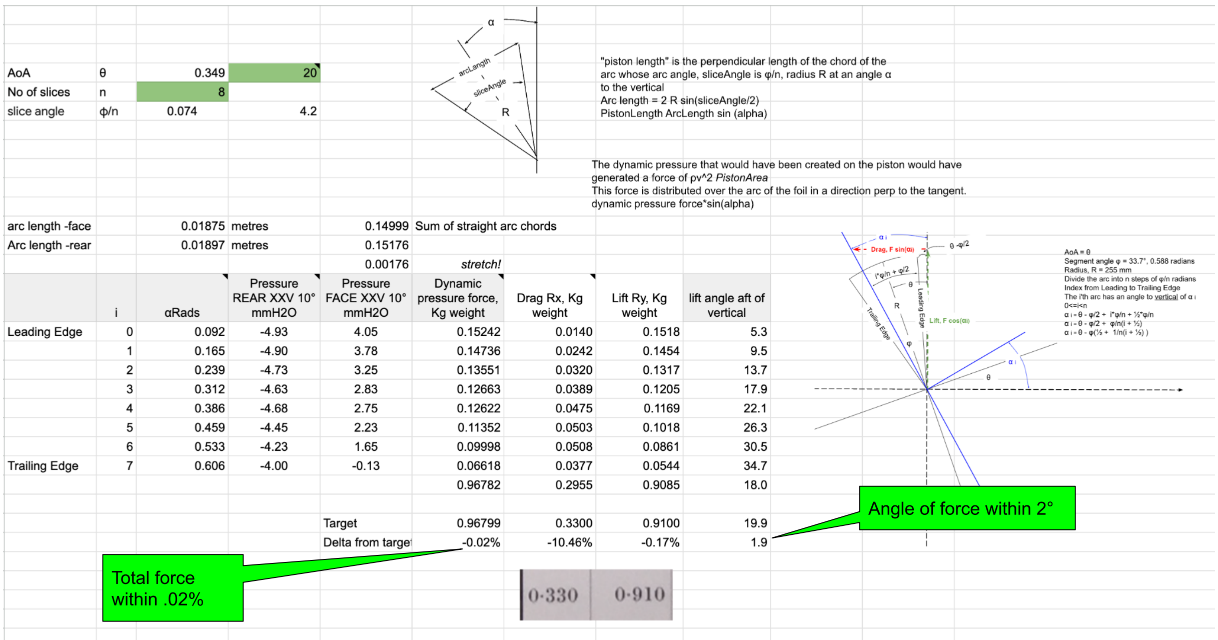


Figure 24 The digitised pressure

The data were transcribed into a spreadsheet and was processed using basic geometry and mechanics to convert the pressures to forces as displayed in Fig 20 below.



AoA	θ	0.349	20					
No of slices	n	8						
slice angle	ϕ/n	0.074	4.2					
arc length -face		0.01875	metres	0.14999	Sum of straight arc chords			
Arc length -rear		0.01897	metres	0.15176				
				0.00176	*stretch!*			
	i	αRads	Pressure REAR XXV 10° mmH2O	Pressure FACE XXV 10° mmH2O	Dynamic pressure force, Kg weight	Drag Rx, Kg weight	Lift Ry, Kg weight	lift angle aft of vertical
Leading Edge	0	0.092	-4.93	4.05	0.15242	0.0140	0.1518	5.3
	1	0.165	-4.90	3.78	0.14736	0.0242	0.1454	9.5
	2	0.239	-4.73	3.25	0.13551	0.0320	0.1317	13.7
	3	0.312	-4.63	2.83	0.12663	0.0389	0.1205	17.9
	4	0.386	-4.68	2.75	0.12622	0.0475	0.1169	22.1
	5	0.459	-4.45	2.23	0.11352	0.0503	0.1018	26.3
	6	0.533	-4.23	1.65	0.09998	0.0508	0.0861	30.5
Trailing Edge	7	0.606	-4.00	-0.13	0.06618	0.0377	0.0544	34.7
					0.96782	0.2955	0.9085	18.0
				Target	0.96799	0.3300	0.9100	19.9
				Delta from target	-0.02%	-10.46%	-0.17%	1.9

Figure 25. The spreadsheet analysis of the digitised isobars

The resulting calculations provided a very close correlation to the forces measured experimentally as displayed in Fig 22.

This provides strong evidence that the aerodynamic force, both its strength and its angle to the foil is **completely determined by, and can be calculated from** the geometry of the foil and the measured pressure differences.

Fig 26 highlights the discovery that the application of physics and maths to the **pressure** measurements yields results that correspond to the **force** measurements.

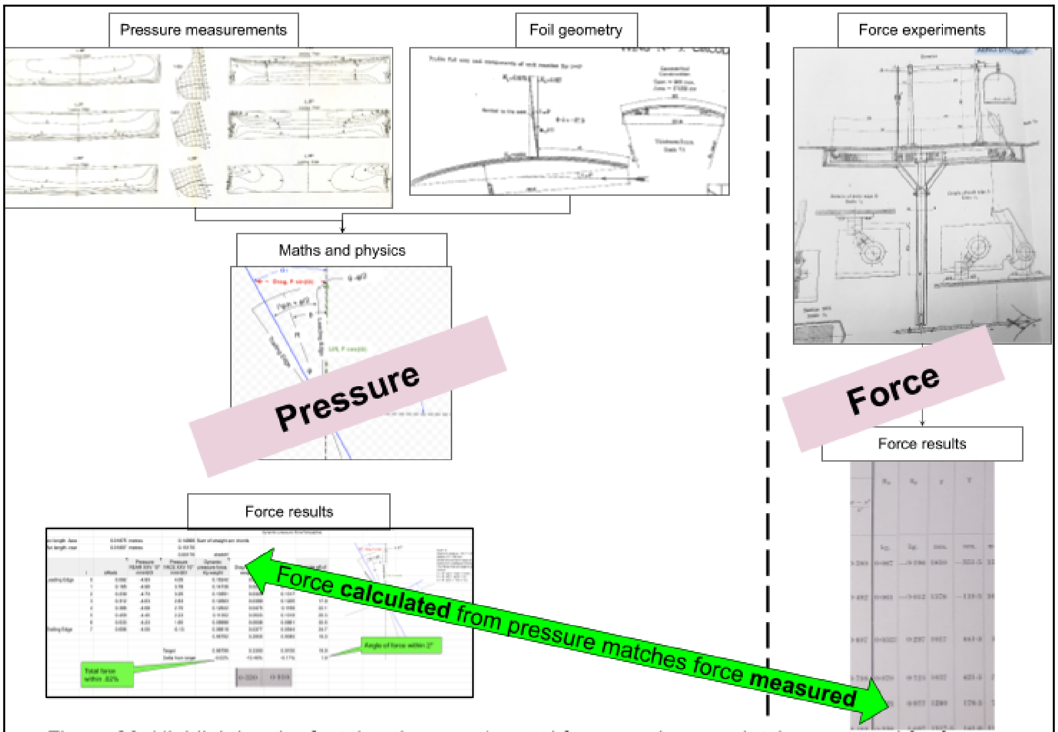


Figure 26. Highlighting the fact that the experimental force can be completely accounted for from the measured pressures, the geometry of the foil and the wind tunnel air speed.

Do the numbers add up?

When we apply the pressure equations to the Eiffel experiment analysed earlier, where the air speed in the wind tunnel was 10 m/s, or Mach 0.03, and the air pressure was 1 atmosphere, or 760 mmHg, we get the following pressure values:

- Front face of the foil, from(8) :

  $p_1 = p_0 (\frac{c_s}{(c_s - c_p)})^{\gamma}$:

  p_1 = 760 *(1/0.97)$^{1.4}$ = 793, or 33 mmHg above 1 atm.
- Rear of the foil, from (9):

  $p_1 = p_0 (\frac{c_s}{(c_s + c_p)})^{\gamma}$:

  p_1 = 760 *(1/1.03)$^{1.4}$ = 729 , or 31 mmHg below 1 atm.

Spreadsheet:

fx = 760 *((1/0.97)^1.4)

	A	B	C
	Front (Cs - Cp)	793.1095225	33.10952255
	Rear (Cs + Cp)	729.1912988	-30.80870125

Figure 27

(The corresponding temperature differences, from (10) , $T_1 = T_0 (\frac{p_1}{p_0})^{(1-\frac{1}{\gamma})}$, based on a temperature of 27°C or 300°K show a change of +3°C and -3°C on the front and rear faces resp.)

Now these values are significantly higher than the values reported by Eiffel of around 3 mmHg on the face and -5 mmHg on the rear, which brings us to the earlier observation:

*"Before addressing the obvious issues of the **angle of the cylinder** to the sail and the direction of the sail through the air, and its relationship with the disturbed air adjacent to the sides of the cylinder, …"*

There are two primary sources of these discrepancies:

1. the foil was oriented at 20° to the wind, not at the 90° of the piston

2. The "cylinders" are not sealed around their sides.

Foil angle

Looking at point 1. If the piston had been tilted to 20° to match the Eiffel experiment in Fig 28 below,

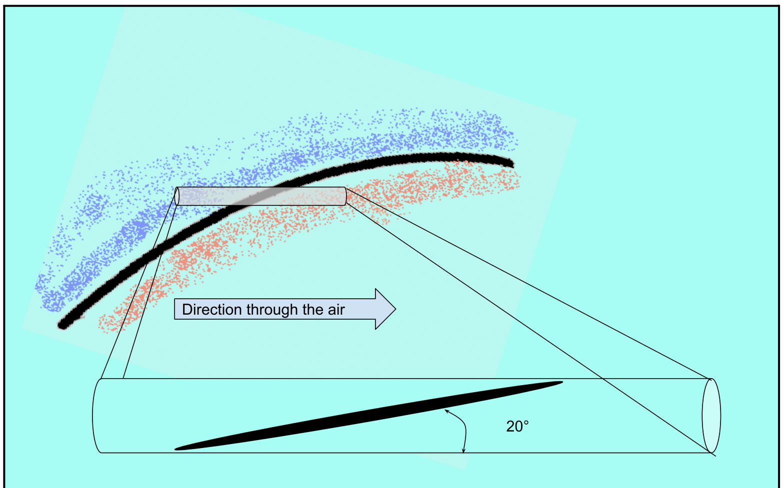


Figure 28. Effect of the 20° inclination of the foil.

then the pressure along the tube would have been distributed over a greater area, reducing the *effective pressure* by a factor of sin(20°), or 0.34 :

face of the foil,

$$31 \times 0.34 = 10.5\, mmHg$$

rear of the foil,

$$-33 \times 0.34 =- 11\, mmHg$$

Sealed cylinder

This is a less tractable problem.

Whilst the cylinders at the centre of the foil are are effectively sealed by the similarly pressurised air around them, cylinders towards the edge of the foil will be exposed to the undisturbed air pressure ahead of the leading edge and behind the trailing edge (Fig 29.) which could easily reduce the pressure by a significant factor

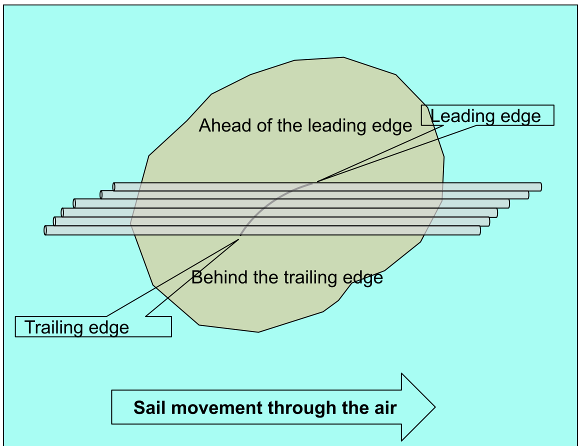


Figure 29. Showing the regions ahead of and behind the foil.

There are also regions around the ends of the plate where we see anomalies in the pressure distribution as illustrated in Fig 30.

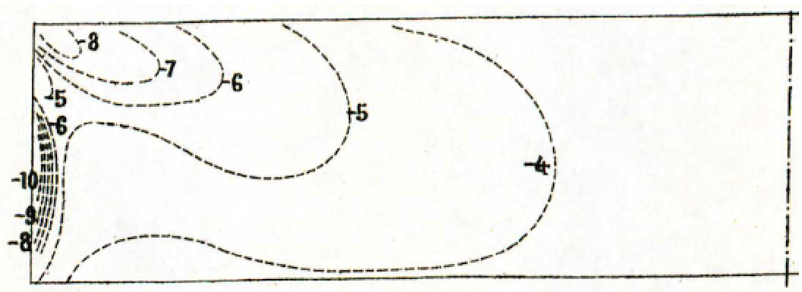


Figure 30. Pressure anomalies at the end of the plate

These deviations from the sealed cylinder implemented in the calculations could easily account for the discrepancy between the calculated (10.5, -11 mmHg) and observed (3, -5mmHg) pressures around the Eiffel plates.

Conclusion

Evidence has been provided for the hypothesis to explain the pressure variations on either side of the moving foil without having recourse to an associated increase or decrease in flow speed.

The direct relationship between the experimentally determined pressure differences and the resultant forces has been demonstrated using geometry, mechanics and classical physics.

The applicability of thermodynamics to the study of these phenomena has been demonstrated.

The approximate accuracy of the results of the thermodynamic approach provides assurance that we are on the correct course of action.

Applicability

The theory has been presented to explain the source of the pressure variation, and thus the aerodynamic force, on a sailboat's sail. It is not claimed that it applies equally to aeroplane wings, bird flight, wind turbines, aircraft propellers, or windmills, which may indicate a direction for further research.

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