Origins of pressure differences around a sail

The adoption of an alternative frame of reference transforms the problem from fluid dynamics to thermodynamics.

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Alan Smith, BSc.
Sydney, Australia
smithalan@bigpond.com

Abstract

Sailboats sail upwind. They use a force, the aerodynamic force, which beneficially exploits the relative motion of the sail through air in a manner that is variously described as mysterious, little understood or so complex that the explanation appears to require a deep understanding of fluid dynamics.

It is well understood that the aerodynamic force derives from pressure differences on each side of the sail, whether it be the low pressure "pull" to leeward or the high pressure "push" from windward. However, a review of the literature reveals that existing theories about the source of those pressure differences are flawed or inadequate.

This paper asserts that the pressure differences are caused only by the relative movement of the sail exerting pressure on air in front of it, and leaving a region of low pressure behind it.

In addition, it asserts the novel position that both of these pressure differences can be explained without recourse to an increase or decrease in flow speed.

Principles from thermodynamics are used to demonstrate the characteristics of the pressure differences.

Introduction

Modern sailboats come in many forms: from 3 metre dinghies, 10 metre club keelboats, 30 metre super-maxies to 100 metre mega sailing yachts. They come with one, two, or many sails made of thin, strong, flexible fabric, supported by one, two, three or more masts.

They all come with a particular facility that is exhibited by even the simplest, single-sailed craft such as a Finn Dinghy or a Laser: they can sail upwind.

They cannot sail directly upwind. No sailboat can sail directly into the wind, but, given two points, A and B in open water, where B is upwind from A, then, without the assistance of tide or current, all sailboats can, by following a zig-zag path, sail from point A to point B.

The challenge for a racing sailor on a course that inevitably contains such upwind legs, is to get from A to B quicker than the competitors.

As an amateur racing sailor the author has learnt many principles of sail and rig trim that are effective in racing. Poorly trimmed sails or poorly tuned rigs make the boat go slower. Better trimmed sails make the boat go faster.

Our instruments can provide feedback by providing Log speed, Apparent Wind Angle (AWA), True Wind Angle (TWA), Velocity Made Good (VMG) etc., but their reliability is dependent on being correctly calibrated and our ability to interpret the numbers.

We make various adjustments and the boat either goes faster, or slower, but we never really know why the adjustment yielded the results.

More often than not the answer will be couched in terms like laminar flow, boundary layers, laminar separation, viscosity, vortices, aerofoil shape etc., all of which come from fluid dynamics, which are totally obscure to the layman sailor and which don't really address the question of when and by how much to apply an adjustment.

The literature of sailing abounds with the classical diagram of the forces on a sailing boat and Fig.1 has been taken as the primary source from Fossati [5] .

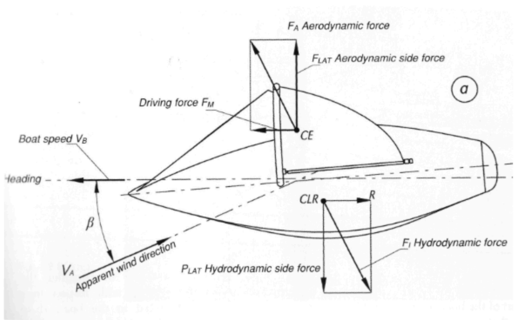

Fig.1 Forces on a sailing boat

The arrows in the diagram indicate the size and direction of the forces acting on the boat.

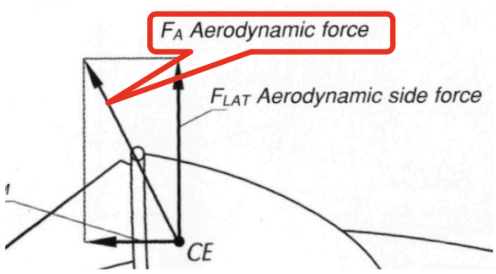

Fig.2 Aerodynamic force

Most of the forces in the diagram were well explained in Fossati's book but the explanation for "F_A Aerodynamic Force" was elusive. From basic physics it is well known that the only ways in which fluid can exert a force on a solid are by applying a pressure over an area, and through friction between the fluid and the solid. Since air is a gas, and a gas is a fluid, and the sail is a solid (if flexible) object, the aerodynamic force has to come from pressure and/or shear stress.

It is also well known that, like friction, shear stress is a dissipative force that converts mechanical energy into thermal energy due to the generation of heat by the interaction between the fluid and the solid surface. It is highly unlikely that any shear stress arising from the flow of air over the sail would make a positive contribution to the aerodynamic force.

So a search was undertaken to identify the nature and source of the pressure differences around a sail that are responsible for the creation of the aerodynamic force.

Literature review

While sailing books may not always fall into the category of scholarly works, many of them approach the subject in a scholarly manner. Sailing is a complex and technical subject that requires a deep understanding of various topics, including navigation, meteorology, sail theory and boat design. As a result, many sailing books are written by experts in these fields who have conducted extensive research and have a wealth of practical experience.

In the search for an explanation of the source of the pressure difference, recommendations were sought from various sources including sailing acquaintances, on-line forums and references. Given the number of titles, the list is by no means comprehensive but there was no deliberate bias in the selection.

What follows is a brief review of the explanation of the source of the pressure variations presented in a sample of the sailing literature about the source of pressure differences around a sail.

Sailing Literature

Fossati [5]

Fossati's book, "*Aero-hydrodynamics and the performance of sailing yachts*", epitomises many of the approaches taken to explain the source of the aerodynamic force.

Fossati is cited in a number of research papers on the aerodynamics of sails by Prof. Richard Flay of Auckland University.

A professor of applied mechanics, Fabio Fossati was selected as a primary source of technical sailing literature as he taught fluid mechanics, naval architecture and mechanics of the sailing yacht on the Master's course in Yacht Design at Milan Polytechnic. He was Research Associate of the International Technical Committee of the Offshore Racing Congress and also a keen sailor.

In chapter 5 "Sailing Boat Aerodynamics Para 5.1 "The aerodynamics of the sail", p. 92, Fossati asserts that::

" …when a sail deflects and affects airflow, because of the link between flow speed and pressure (see Appendix 1) an area of weak high pressure is created to windward and a suction area is created to leeward of the sail."

In Appendix 1, "Elements of Fluid Mechanics", the existence of these high and low pressure regions is clearly demonstrated experimentally in fluids, both liquid and gas, flowing through tubes and around foil sections.

The existence of these pressure variations is not a revelation.The source of these pressure variations is attributed as follows:

"These pressure variations are attributable to a variation in dynamic pressure induced by a flow speed variation. It is in fact evident that where the section where the flow passes narrows, because of the conservation of fluid flow, there must be an acceleration of the flow which must correspond with a reduction in pressure."

It is the circular nature of this argument that is at the heart of the mystery: on p. 92, the deflection of the airflow by the sail influencing the flow speed is the source of the pressure variations, and now in Appendix 1, we find that the pressure variations must be due to an acceleration of the flow.

In the first, the pressure variations are due to flow speed and in the second, the variation in flow speed is due to pressure variations.

Bernoulli's Law is invoked to explain the link between the accelerating flow and the pressure variations and from there, the fluid dynamics arguments are based on the existence of an accelerated airflow.

Fossati presents no evidence to support the existence of an accelerated airflow over an aerofoil.

Whidden and Levitt [9]

Tom Whidden is president and CEO of North Technology Group, which includes North Sails. Whidden is an America's Cup sailor who has won the Cup three times: 1980,1986-87, and 1988, primarily as tactician for Dennis Conner. Whidden is in the America's Cup Hall of Fame. He is the author of three books on sails and sailing including "*The Art and Science of Sails Revised Edition*" which is referenced below.

p. 57:

'*A sail is like an airplane wing.' If you missed that lesson early in your sailing career, you likely heard its antecedent in sixth-grade science: An airplane flies because air passing over the curved upper surface of its wings has to travel a longer distance than the air passing under the flat lower surface. And since it has to go farther, it has to go faster to reach the trailing edge at the same time as its brother particle. This difference in distance causes a difference in speed that causes a difference in pressure-low pressure on the upper side of the wing (lee side of the sail) and high pressure on the lower side of a wing (wind-ward side of a sail)*".

This is an example of the "Equal Transit Time " explanation, a common misconception which is often used to explain the increase of the speed of the air speed flowing over a wing or a sail as required by Bernoulli's Principle to generate a low pressure.

Jobson [6]

Gary Jobson has won ten national one-design sailing titles, the America's Cup, and innumerable ocean races. In 1999 he received the U.S. Sailing Association's most prestigious award, the Nathanael G. Herreshoff Trophy. In 2003 Gary was inducted into the America's Cup Hall of Fame. He is an editor at large for Cruising World and Sailing World and has been ESPN's sailing commentator since 1985.His book "*Championship Sailing* " is referenced below:

P. 166 :

"*No matter what sail you may be flying - mainsail, genoa, spinnaker, or staysail - they all work on the same principle: wind bends around the sail, causing a vacuum on the forward leeward side. The boat moves forward to fill this vacuum.*"

This is an interesting view but, unfortunately he takes this no further to explain the nature of this phenomenon or to refer to the existence or significance of the high pressure to windward.

Dedekam [3]

Ivar Dedekam has no great CV but has written a small but highly regarded handbook, "*Illustrated Sail & Rig Tuning*".

p. 3:

"*The same happens when air flows along a sail (or an airplane wing). The shape of the sail forces the airflow on the leeward side to take a longer path than on the windward side. Therefore the air has to* *increase its velocity on the leeward side of the sail resulting in a lower pressure than on the windward side. (Bernoulli's principle states that an increase of velocity in a fluid flow gives a pressure decrease.)*"

Once again the disproven "equal transit time" theory is used to explain the pressure variation.

Melges[7]

Harry C. "Buddy" Melges Jr. is a competitive sailor. He has earned national and international championships in several classes in conventional sailing and ice-boating. His book "*Sailing Smart*" is referenced below:

P. 73:

"*Every sail gets its driving force from its shape, and this power is controlled by the amount and location of a sail's draft, or camber. Never allow the air flow to stall or separate from the sail's surface (unless you become overpowered) because that reduces the driving force of the sail. Always keep in mind that it is the amount and the velocity of the air that flows across the underside of your sails that determine how much power you are going to get from those sails. Maintaining this smooth flow applies to all points of sail: on the wind as well as running free.*"

Simple and direct: it's the shape of the sail that provides the driving force. However, there is no attempt to explain how this occurs.

Marchaj [8]

C.A. Marchaj was a visiting research fellow at the Department of Aeronautics and Astronautics, Southampton, England, where he held a Master's Degree in the Faculty of Engineering and Applied Science. A former Polish champion of the Finn class, he was a chartered engineer, a glider pilot, and a member of the Royal Institution of Naval Architects. His book, "*Aero-Hydrodynamics of Sailing*" is widely referenced as an authority on the subject.

P. 180:

"*A little reflection however shows that Bernoulli's theorem locates the region of higher pressure in places where the free motion of fluid is retarded. Since pressure may be regarded as a form of energy and Bernoulli's equation indicates that a balance is maintained between energy arising from the motion and that from the pressure in all parts of the stream; it becomes rather obvious and inevitable law that what has been lost in one form of energy must be recovered in another form (Ref 2.5).*"

Marchaj ascribes the pressure difference to a velocity difference but fails to explain the source of the velocity difference.

Aerodynamics literature

Since these sailing books failed to deliver a satisfactory answer and many of them invoked aerodynamics or description of a sail being like a wing, a search of the aerodynamics literature was undertaken.

However, after an extensive search of the literature, none could be found that discussed sails or provided any information about the source of the pressure differences around a sail.

Summary

Whilst this is by no means an exhaustive review of the available literature, and indeed is a small subset of the books, articles and websites that have been reviewed, it is a representative cross-section of the way that the source of the pressure variation is addressed.

The sailing literature leaves a gap in the explanation of the source of the pressure difference which remains unfilled by the aerodynamics literature.

It is apparent that there is no consensus on the subject and that there is no satisfactory documented explanation for the source of the pressure difference.

Hypothesis

The hypothesis is that the pressure variations are caused only by the movement of the sail through the air, compressing the air to windward and leaving a relative void to leeward.

This hypothesis asserts that pressure differences, both increased and decreased, can be generated in air without reference to any associated increase or decrease in flow speed.

The hypothesis is tested using:

- an alternative frame of reference with the sail moving in still air rather than the wind blowing over the sail
- a thought experiment using a foam of idealised soap bubbles to represent the volume of air through which the sail moves.

Frame of Reference

The title of this section may tempt the experienced reader to skip over the details, but be assured that it leads to the heart of the matter and will reward the reader with a novel insight into the solution of the problem.

Let us pause to consider the wind. Meteorological services describe the wind using various parameters to convey its characteristics accurately. These parameters include:

Wind Direction: Wind direction indicates the compass direction from which the wind is blowing. It is typically reported descriptively as Nor'easter, Southerly or in numerical compass bearings like 045° .

Wind Speed: Wind speed refers to the rate at which air is moving past a given point. It is commonly measured in units such as metres per second, kilometres per hour or knots (nautical miles per hour). Maritime services also use the traditional Beaufort scale of Forces to describe the wind speed.

As sailors, we are aware of the effect of the wind on our sails, and are aware that the perceived wind is different to the reported wind, and that is due to the direction of travel relative to the wind. When sailing upwind, the perceived wind speed is always greater than when sailing downwind in the same breeze. This of course is well understood as the apparent wind, which is different from the true wind that is measured relative to a fixed point. And the difference between the true wind and the apparent wind is due to a third factor: the boat's speed and direction through the water.

So we are familiar with these two frames of reference: the one from which the Met office reports the wind, which is based on the wind's speed and direction in relation to a fixed point, and that which we experience on the boat, which is moving in relation to the land. These two frames are related by the boat's movement in relation with the land or the water (assuming for the moment there is no significant tide or current).

In both of these frames of reference, it is the air that is perceived to be in motion. Wind, is, of course just that: air in motion, and it is this perception that has given rise to the presence of the suffix "dynamics" in the name of the technologies of aerodynamics and fluid dynamics, that is widely invoked to explain the mechanics and kinematics of not only sailing, but aeronautics, propellers and wind turbines.

But there is a third frame of reference: one in which the motion is not of the air, but that of the sail itself, and it is in that frame of reference that we explore the source of the pressure differences that generate the force that propels a sailboat.

In this frame of reference we can consider the air, in its classical form, as a continuum in a steady state. Just as we can consider the water through which the boat is moving as a continuum in a steady state being disturbed by the passage of the boat causing waves and ripples, we can consider the sail as moving through a continuum of air, generating waves and ripples in the air. In this case, of course these waves and ripples are disturbances in the thermodynamic properties of the air: its pressure, density and temperature. When considering the water as being in a steady state, we don't have to consider the velocity of the water as it is the boat that moves through the water. Similarly , we don't have to be troubled by the perplexing conundrum of the velocity of the air when we consider the sail moving through the (steady state) air.

The complexities of concepts such as individual air particle properties, including size, position, velocity, mass and momentum, can be effectively bypassed within this framework. Here, the focus shifts to the thermodynamic behaviour of air, aligning the analysis with the established principles of gas theory. This approach allows us to treat air as a continuous medium, simplifying the problem and highlighting the role of pressure, density, and temperature variations in sail propulsion.

In this paper, we propose a novel perspective for analysing sail behaviour. By considering the sail moving through a static "air continuum", we can focus on the sail's interaction with the thermodynamic properties of air (pressure, density, temperature). This approach departs from traditional aerodynamics and aligns well with the principles of thermodynamics and gas theory. This shift in perspective offers a valuable new lens for understanding the complex forces at play in sail propulsion which is developed in the following sections.

It is well established that a sailing yacht is able to sail with the wind blowing from a direction ahead, so the explanation commences with Fig. 3, a diagram of a yacht sailing upwind. The apparent wind angle is the angle between the sail chord and the apparent wind direction. (Fossati [5]).

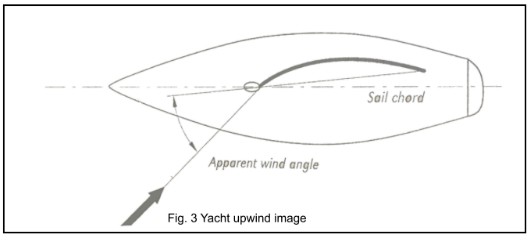

Fig. 3 Yacht upwind image

To develop the theory, the image has been flipped and rotated in Fig. 4 to bring the apparent wind direction to the horizontal.

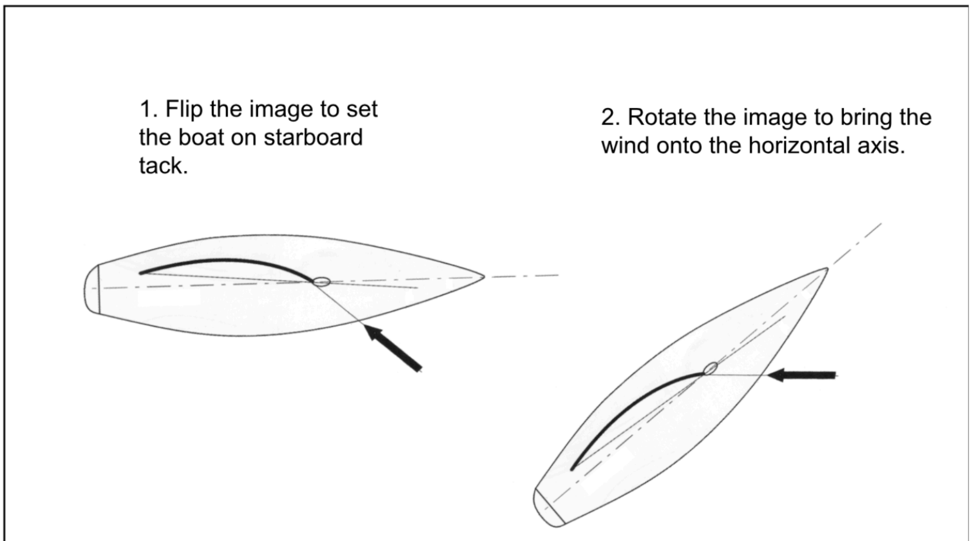

Fig 4. Transposed image

The sail is now considered in the frame of reference introduced earlier, of the apparent wind, with the sail moving through the wind instead of the wind moving over the sail.

In either frame of reference, the laws of physics apply. Strictly speaking, as long as the frames of reference are not accelerating, any one frame of reference is as good as another.

In this new frame of reference, the sail is moving in relation to the stationary air instead of the wind moving in relation to the stationary sail. Fig. 5

And since, in this frame of reference, the wind is stationary, we will simply refer to it as air.

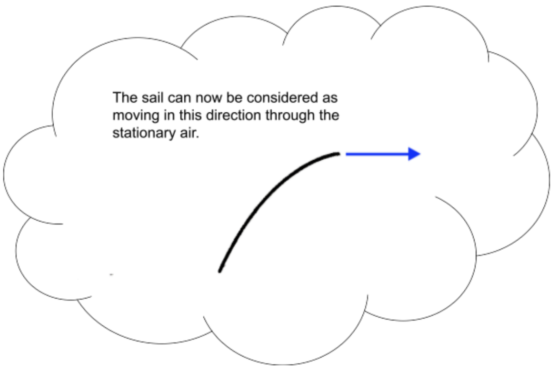

Fig. 5 Sail moving relative to still air

The significance of this change of perspective is that it emphasises the point that this is no longer a problem of fluid dynamics. If the sail is moving through still air, it may disturb the air through which it is moving, but the disturbance only occurs due to the sail's passage through the otherwise tranquil air.

Sail Movement

Let's now consider what happens as the sail moves through the air from position A to position B. Fig 6.

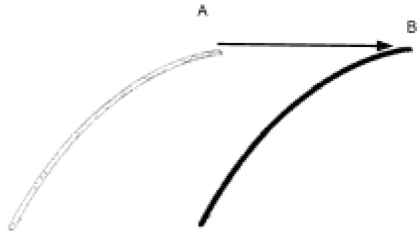

Fig. 6 Sail moving A to B

Let's call the area between the sail at the two positions the "swept area", as like a broom, the sail sweeps over the area. Fig. 7

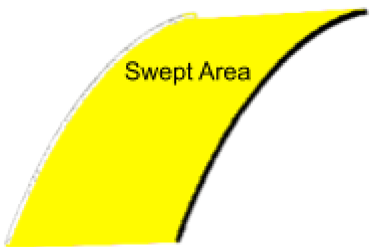

Fig. 7 Swept Area

And let's not forget that we have been looking at a 2-dimensional representation, i.e. a cross section of the sail Fig 8. (Fossati [5]).

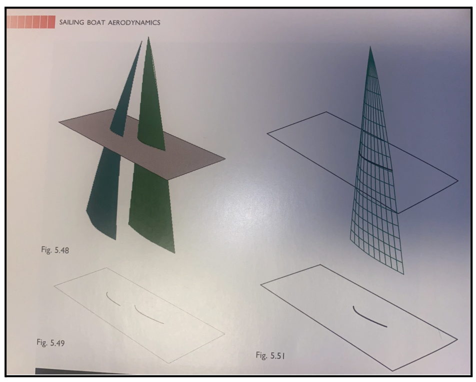

Fig. 8. Cross section through the sail

The swept area represents the cross section through a **volume** of air, and so we will refer to it as the "swept volume". Fig. 9

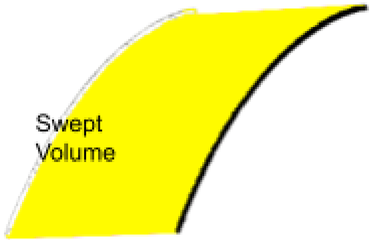

Fig. 9. Swept Volume

Thought Experiment

You are now invited to participate in a "thought experiment" in which the air through which the yacht is sailing is filled with small soap bubbles. This is not an ordinary foam of soap bubbles of different sizes, whose film is made up of soapy water which is fragile and wet, but one that consists of equal sized idealised bubbles whose film is composed of a material that is thin, strong, light and infinitely flexible. This film also exhibits the same characteristic that is shared by all liquids including the soapy water that creates bubbles, namely surface tension that binds the film and provides the pressure that causes free-floating bubbles to take on the familiar spherical shape that encloses a given volume with the smallest surface area.

When bubbles pack together, like in foam, the space will be filled with bubbles and surprisingly a lot of research has been done to investigate the shape these bubbles will form. Searching "Weaire Phelan structure" will take you down a rabbit hole of space-filling convex polyhedra and Kelvin structures, like in Fig 10. but suffice it to say that the bubbles so formed will not be the familiar spheres of free-floating bubbles. If the bubbles are of equal size, they will form a tesselated mesh of polyhedrons with flat faces, and straight sides.

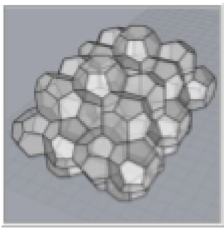

Fig 10 Tessellated polyhedra

In the following figures, the bubbles are represented by circles and represent a cross-section through a 3-dimensional region.

In Fig 11, the bubbles in the swept volume have been coloured red. These are the bubbles that would normally occupy the region swept by the sail as it moves from position A to position B

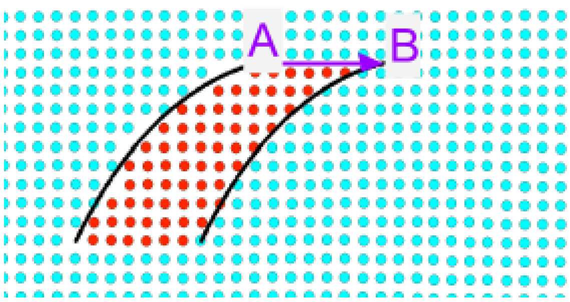

Fig. 11 Red bubbles in swept volume

All the red bubbles are swept by the sail into the space that was previously occupied by cyan bubbles. We know that two bubbles are unable to occupy the same space and time, so they must become compressed and flow out into the region ahead, forcing most of the bubbles in the region ahead to also become compressed.The bubbles in the swept volume become compressed along with the bubbles that previously occupied the space as well as some of the bubbles further upwind.

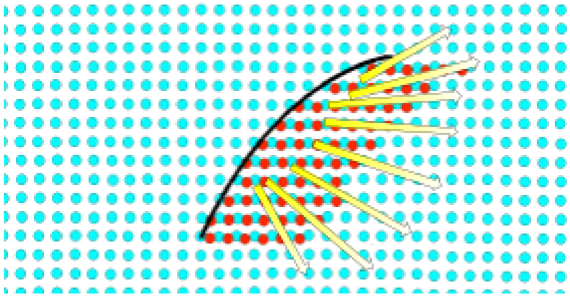

Fig.12 Compressed red bubbles

Eventually, as the bubbles are pushed further outwards they become decreasingly compressed until at some distance to windward, their pressure difference from the original bubbles becomes insignificant.

Attention is now turned to the swept volume to leeward or behind the sail in Fig. 13. It is apparent that bubbles in the region behind the sail will now have to move in to fill the void that is left as the sail moves forward. Since bubbles can't be created to fill this void, they are drawn in from the surrounding region, so the same number of bubbles will now have to occupy a larger volume. With the same amount of air filling a larger space, these bubbles will now experience air pressure lower than the surrounding region.

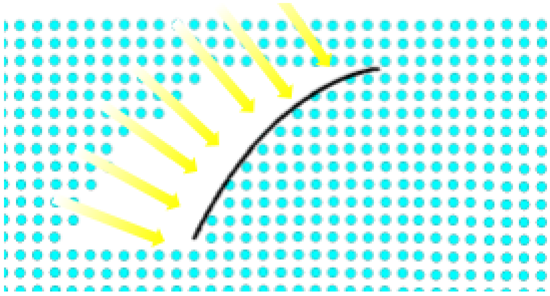

Fig. 13 Swept volume to leeward.

So now this region of high pressure can be seen in front of the sail, with this region of low pressure behind the sail. Fig. 14

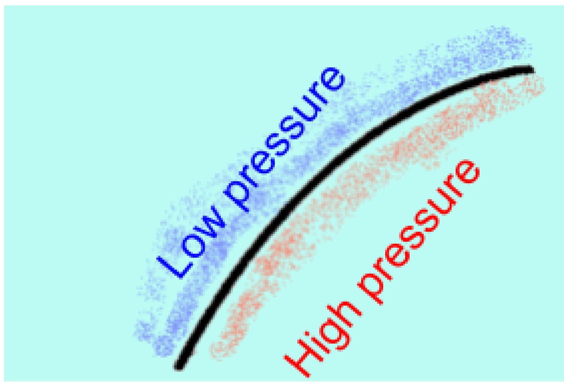

Fig. 14 pressure distribution

The pressure regions

The question now arises as to the characteristics of these high and low pressure regions: their size, shape and intensity and leads the discussion into the consideration of thermodynamics. Whilst thermodynamics is a general science that deals with the relationships between heat, work, temperature, and energy, its application to real-world problems is primarily through the study of the nature of gases, such as the air around the sail in which the pressure regions occur.

The behaviour of gases is governed by the ideal gas law, which states that the pressure of a gas is proportional to its temperature and inversely proportional to its volume. This law can be used to predict the behaviour of gases in a variety of situations, such as in engines, refrigerators, and air conditioners.

In addition to the ideal gas law, thermodynamics also provides insights into the behaviour of other types of systems, such as liquids and solids. However, the study of gases is often the most straightforward way to apply thermodynamics to real-world problems.

The ideal gas law can be used to calculate the pressure, temperature, and volume of a gas.

The kinetic theory of gases can be used to explain the behaviour of gases at the molecular level.

The ideal gas law relates the pressure, temperature and volume of a quantity of gas through the equation PV=KT, where

- P is the pressure of the gas
- V is its volume,
- T is it temperature
- and K relates to the amount of the gas under consideration.[1]

The ideal gas law is a simplified model of the behaviour of gases. It assumes that gas molecules have no volume and do not interact with each other. However, air molecules do have a small volume and do interact with each other.

This deviation from the ideal gas law is negligible at the temperatures and pressures experienced around a yacht's sail[2].

The Volume challenge

The challenge in applying thermodynamics to the problem of the sail is that whilst the pressure (P) and temperature (T) of the air around the sail is quite easily identified, the question of volume (V) presents a challenge to the use of thermodynamics in this situation.

However, consideration of the speed with which a disturbance in the pressure, temperature or density propagates through the air makes the problem less intractable. That speed is, of course, the speed of sound in the air.

So the passage of the sail through the air is influenced by the pressure in the air that has been disturbed by its passage.

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[1] PV = nRT is the usual formulation, but since R is the Gas Constant and "n" is the number of moles of gas under consideration, their combination into a single variable K related to the amount of gas is accepted practice.

[2] Cengel [2] p. 726: "The temperature of air in air-conditioning applications ranges from about −10 to about 50° C. In this range, dry air can be treated as an ideal gas with a constant cp value of 1.005 kJ/kg·K [0.240 Btu/lbm·R] with negligible error (under 0.2 percent). … The atmospheric air can be treated as an ideal-gas mixture whose pressure is the sum of the partial pressure of dry air P_a and that of water vapour P_v"

Consider the following Fig 15.

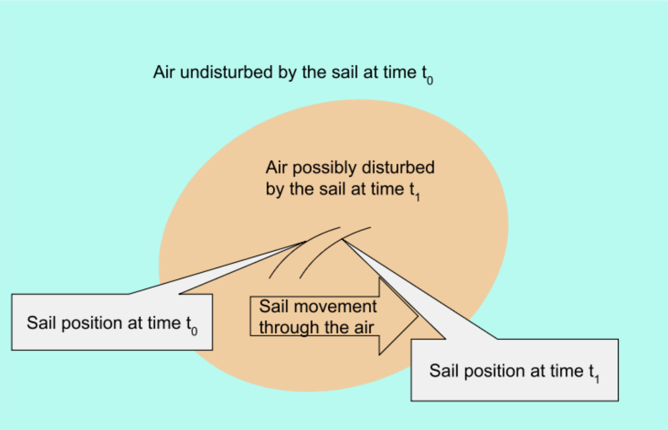

Figure 15. Region of air disturbed by the sail

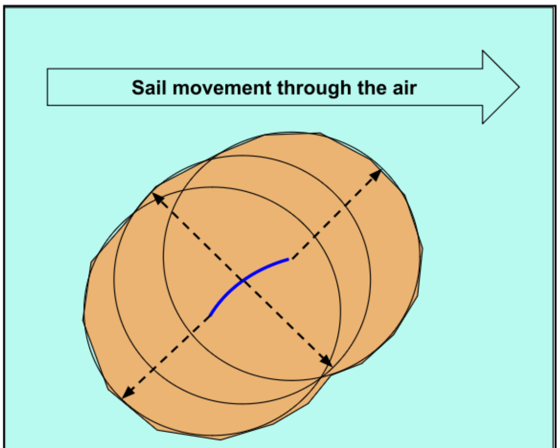

Figure 16. Boundary of disturbed region.

At any particular instant in time, t_1, there is a region beyond which the speed of sound prevents any changes to pressure, density or temperature disturbance from the passage of the sail through the air at time t_0. The boundary of the possibly disturbed air is determined by the distance that the disturbance can travel from the sail in time $t_1 - t_0$ which we shall call "delta t" or Δt.

The distance is shown as the equal-length dotted arrows in Fig 16 which shows the boundary around the region.

The length of the arrows is $c_s \times \Delta t$, where c_s is the speed of sound in the undisturbed air.

It should be noted that the above figures are not intended to indicate any scale, since the speed of the sail through the air is much lower than the speed of sound in air. A 20 kt apparent wind speed (typical of a yacht doing about 7 kts upwind in a 15 kt wind) relates to about Mach 0.03, or .03 times the speed of sound.

Nor does the colouring of the region indicate anything about the distribution of pressure, temperature or density within the region. These features are discussed in the following section.

It should also be noted that this is a 2-D representation - effectively a horizontal cross-section through the sail.

The piston analogy

To explore the way that the sail generates the pressure differences, we return to Fig 14 and consider the long narrow cylindrical region that passes through the sail as visualised in Fig 17.

The ends of the tube are open to the region of air that is undisturbed around the sail as discussed in the previous section and the section of sail can be considered as a piston moving through this cylinder.

Before addressing the obvious issues of the angle of the cylinder to the sail and the direction of the sail through the air, and its relationship with the disturbed air adjacent to the sides of the cylinder, its consideration from a thermodynamics perspective will yield an valuable and unexpected feature which is explored below.

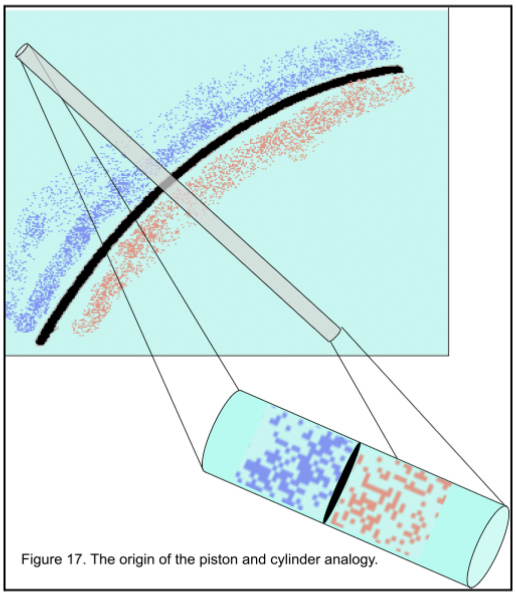

Figure 17. The origin of the piston and cylinder analogy.

A piston pushing a fluid through a cylinder is a construction that is widely studied in the literature of thermodynamics, particularly when discussing the speed of sound and Mach number.

As the piston progresses through the fluid, a pressure wave is generated that has a well defined wave front (Fig 18.) where there is an abrupt change in the pressure and density of the fluid, and this wave front propagates at the speed of sound in the stationary fluid. Cengel [2] p. 843

It is important to recognise that this wave front is not a "shock wave" generated by a supersonic aircraft or speeding bullet, because this pressure wave will be generated by a piston, or the sail, that is moving through the air at well below the speed of sound. The 20 kt wind speed used in Eiffel's experiments [4] and experienced while sailing equates to about 3% of the speed of sound, or Mach 0.03.

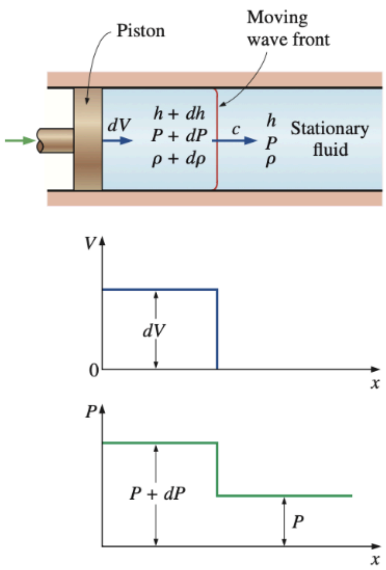

Figure 18. Example of a cylinder and piston used in thermodynamics. Cengel[2] p 843

Consider the following example of a piston moving in a long tube open at each end to the undisturbed air described above. The piston is shown in Fig 19 below at time t_0 and at a short time later, at time t_1.

The air ahead of the piston is being compressed and the air behind the piston is being expanded by the movement of the piston.

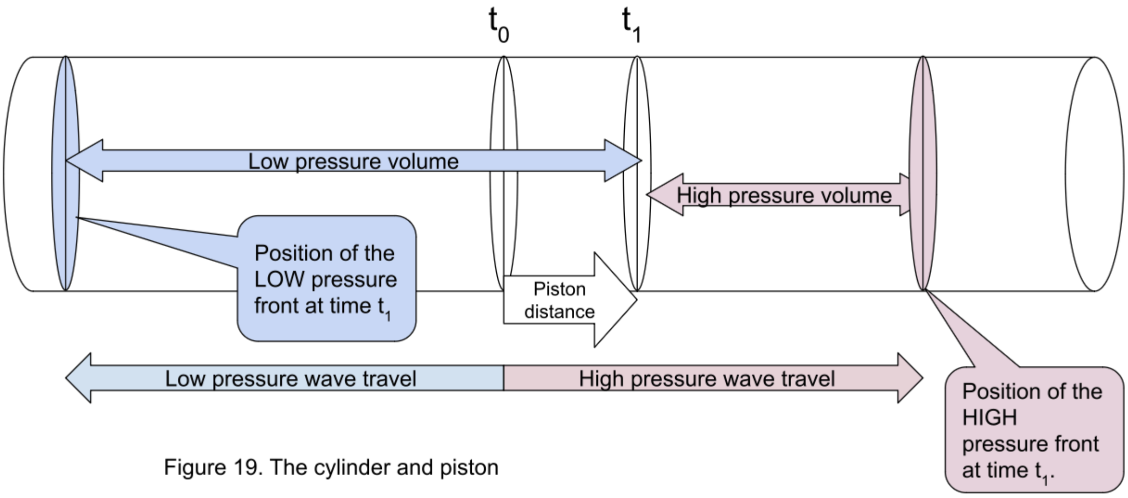

Figure 19. The cylinder and piston

We can calculate the pressure ahead of the piston moving at speed c_P, after time Δt using the adiabatic process as follows:

In an adiabatic expansion $pV^{\gamma} = constant$ Blundel[1]l p. 117 (1)

Where

p is the pressure of the gas

V is the volume of the gas

γ (gamma) is the adiabatic constant ($\frac{C_p}{C_V}$)which for air is 1.4

So when a quantity of gas is subjected to an adiabatic expansion, from pressure p_0 and volume V_0 to pressure p_1 and volume V_1, we have

$$p_1 V_1^{\gamma} = p_0 V_0^{\gamma} \qquad (2)$$

p_0 is the known undisturbed air pressure

V_0 is the the cross-section area of the piston (A) x the distance the pressure front travels in time t, so

$$V_0 = A\, c_s \Delta t \qquad (3)$$

Where

A is the cross-section area of the piston

c_S is the speed of sound in the undisturbed air

Δt is the time interval

Substituting (3) into (1)

$$p_0 V_0^{\gamma} = \text{constant} = p_0 (A\, c_s \Delta t)^{\gamma} \qquad (4)$$

V_1 is V_0 minus the volume the piston sweeps through in time Δt,

so $V_1 = V_0 - A\, c_P \Delta t$ (5)

Substituting V_0 from (3) into (5)

$V_1 = A\,\Delta t\,(c_s - c_P)$ (6)

Substituting V_0 from (3) and V_1 from (6) into (2)

$$p_1 = \frac{p_0(A\,c_s\Delta t)^{\gamma}}{(A\,\Delta t\,(c_s - c_p))^{\gamma}} \quad (7)$$

The $(A\,\Delta t)^{\gamma}$ term cancels out top and bottom yielding:

$$p_1 = p_0\left(\frac{c_S}{(c_s - c_P)}\right)^{\gamma} \quad (8)$$

A simple examination of the ($c_s - c_P$) term relating to the volume ahead of the piston for will demonstrate that the pressure **behind** the piston is given by:

$$p_1 = p_0\left(\frac{c_S}{(c_s + c_P)}\right)^{\gamma} \quad (9)$$

This means that the pressure in front and behind the pistons is **independent of the time interval,** Δt, over which the pressure is measured.

This is a significant conclusion because it demonstrates that the pressure generated by the movement of the piston can be derived directly from the speed of the piston relative to the speed of sound in the undisturbed air whose pressure is known.

The "volume challenge" problem has been resolved.

Conclusion

By changing the perspective, or frame of reference, from one in which the wind is perceived to be moving over the sail to one in which the sail is moving through a static continuum of air, we have restated the problem as one purely of thermodynamics and gas theory.

The theory has been presented to explain the source of the pressure variation, and thus the aerodynamic force, on a sailboat's sail. It is not claimed that it applies equally to aeroplane wings, bird flight, wind turbines, aircraft propellers, or windmills, which may indicate a direction for further research.

Many questions remain to be answered, and it is hoped that the adoption of this novel approach may stimulate research into how, for example, the shape of the sail influences the size and direction of the resulting aerodynamic force. After all, it is the experience of the sail trimmers that the shape of the sail, influenced by tension in the many control lines (sheet, halyard, downhall, etc., etc.) directly influences the speed and pointing performance of the yacht.

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